# 1998 MATH Challenge

1. Find the length of the shortest path from the point (3,5) to the point (8,2) that touches both the $x$ and $y$ axes.

2. A bubble-chamber contains three types of sub-atomic particles: 1998 particles of type X, 2002 of type Y, 2003 of type Z. Whenever an X and Y particle collide, they both become Z particles. Likewise, Y and Z particles collide and become X particles, and X and Z particles become Y particles upon collision. Certainly the total number of particles will never change. Is it possible that they can evolve so that only one type of particle is present?

3. Two towns, A and B, are connected by a road. At sunrise, Pat begins biking from A to B along this road, while simultaneously Dana begins biking from B to A. Each person bikes at a constant speed, and they cross paths at noon. Pat reaches B at 5 p.m. while Dana reaches A at 11:15 p.m. When was sunrise?

4. A random number generator outputs integers from the set $\{1,2,3,4,5,6,7,8\}$, with each of the eight choices equally likely. If ten such random integers are created, what is the probability that their product is one more than a multiple of 8?

5. A bug is crawling on the coordinate plane from (7,11) to (-17,-3). The bug travels at constant speed one unit per second everywhere but quadrant II (negative $x$ and positive $y$ coordinates), where it travels $\frac{1}{2}$ units per second. What path should the bug take to complete her journey in minimal time?

6. Show that the roots of the polynomial equation

$$x^{1998} - 2x^{1997} + 3x^{1996} - 4x^{1995} + \ \ldots \ - 1998x + 1999 = 0$$

are not all real.

7. A **great circle** is a circle drawn on a sphere that is an "equator;" *i.e.*, its center is also the center of the sphere. There are $n$ great circles on a sphere, no three of which meet at any point. They divide the sphere into how many regions?

8. For positive integers $n$, define $S_n$ to be the minimum value of the sum

$$\sum_{k=1}^{n} \sqrt{(2k-1)^2 + a_k^2} \ ,$$

as the $a_1, a_2, \ldots, a_n$ range through all positive real values such that

$$a_1 + a_2 + \ \ldots \ + a_n = 17$$

Find $S_{1998}$.

9. Let $f\colon R \to R$ be continuous, with $f(x) \quad f(f(x)) = 1$ for all $x \quad R$. If $f(1000) = 999$, find $f(500)$.

10. Prove that $\dfrac{\sin x}{x} = \displaystyle\prod_{n=1}^{\infty} \cos\left(\frac{x}{2^n}\right)$.